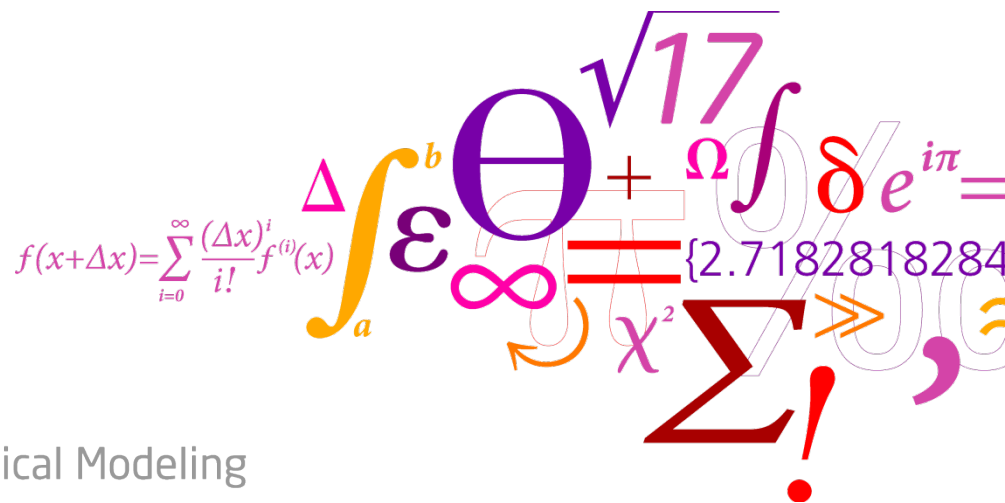


Tools for decision making under uncertainty in electricity markets

Lecture 1: “When uncertainty matters”

Juan Miguel Morales
Salvador Pineda



Outline

1. Optimizing under uncertainty
2. Decision making under uncertainty in electricity markets
3. Solution approaches
4. Example

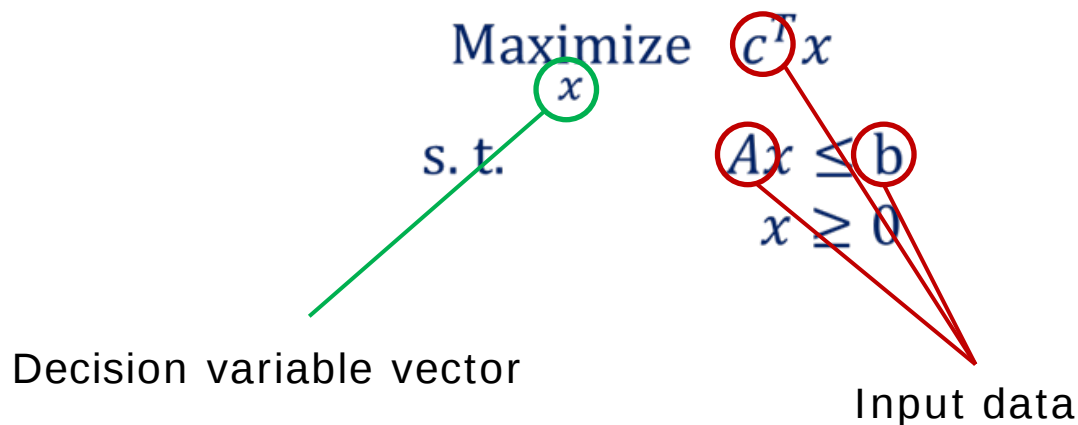
Optimizing under Uncertainty

- Decision problems are often formulated as optimization problems
- Consider the following linear optimization problem:

$$\begin{array}{ll} \underset{x}{\text{Maximize}} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0 \end{array}$$

Optimizing under Uncertainty

- Decision problems are often formulated as optimization problems
- Consider the following linear optimization problem:



How to find the *best* value of “ x ”?

- If A, b and c are perfectly known $\Rightarrow$ deterministic solution algorithms
- Any suggestion?

$$\begin{array}{ll} \underset{x}{\text{Maximize}} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0 \end{array}$$

How to find the *best* value of “x”?

- If A, b and c are perfectly known $\Rightarrow$ deterministic solution algorithms
- Simplex method

$$\begin{array}{ll} \underset{x}{\text{Maximize}} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0 \end{array}$$

How to find the *best* value of “ x ”?

- But what if...
 - ✓ A, b and/or c depend on the realization λ_ω of a certain random parameter λ , i.e. $A(\omega) = A(\lambda_\omega), b(\omega) = b(\lambda_\omega)$, and/or $c(\omega) = c(\lambda_\omega)$
 - ✓ Decision vector x needs to be determined before λ is disclosed

$$\begin{array}{ll}\text{Maximize} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0\end{array}$$

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$$\begin{array}{ll}\text{Maximize} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0\end{array}$$

How can we guarantee feasibility?

How to find the *best* value of “ x ”?

- But what if...

- ✓ A, b and/or c depend on the realization λ_ω of a certain random parameter λ , i.e. $A(\omega) = A(\lambda_\omega), b(\omega) = b(\lambda_\omega)$, and/or $c(\omega) = c(\lambda_\omega)$
- ✓ Decision vector x needs to be determined before λ is disclosed

$$\begin{array}{ll} \underset{x}{\text{Maximize}} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0 \end{array}$$

A family of random variables: $f(x, \omega) = c(\omega)^T x$
Which one is the best?

How to find the *best* value of “ x ”?

- But what if...

- ✓ A, b and/or c depend on the realization λ_ω of a certain random parameter λ , i.e. $A(\omega) = A(\lambda_\omega), b(\omega) = b(\lambda_\omega)$, and/or $c(\omega) = c(\lambda_\omega)$
- ✓ Decision vector x needs to be determined before λ is disclosed

$$\begin{array}{ll} \underset{x}{\text{Maximize}} & c^T x \\ \text{s. t.} & Ax \leq b \\ & x \geq 0 \end{array}$$

How can it be reformulated so that it can be processed by deterministic routines?

Optimizing under Uncertainty ...

... encompasses models, methods, and decision theory concepts to deal with feasibility, optimality and solution procedures in optimization problems involving uncertain data.

Stochastic Programming Community Home Page

<http://stoprog.org/>

Decision Making in Electricity Markets

- Uncertainty is present in most decision-making problems faced by electricity market agents
- Examples?

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- Examples?
 - ✓ Producers/retailers/consumers: Trading and risk management
 - ✓ TSO/MO: Market-clearing procedures
 - ✓ Increased uncertainty due to the growth of stochastic producers (in particular, wind generation)
 - ✓ Generation and transmission capacity investments

Decision Making in Electricity Markets

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Solution approaches



Stochastic programming
with recourse



Chance-constrained
stochastic programming

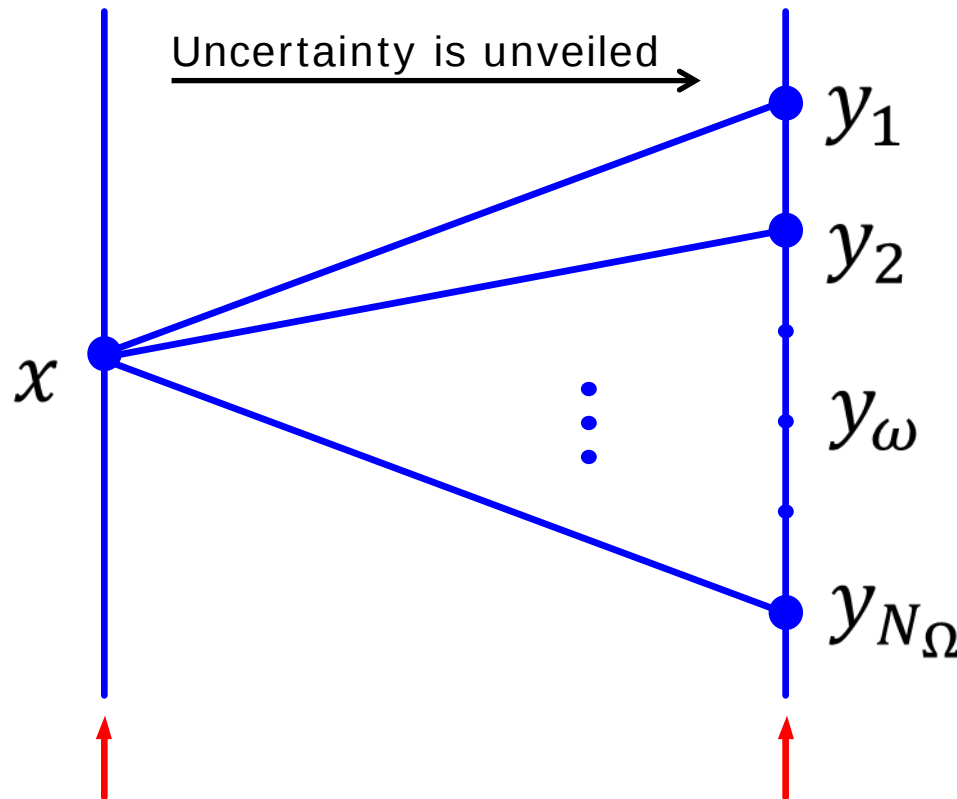


Robust optimization

Stochastic programming with recourse

- Two types of decisions:
 - ✓ Here-and-now decisions (X): made before the uncertainty is disclosed
 - ✓ Wait-and-see or recourse decisions (Y): made once the uncertainty is revealed
 - o They compensate for any bad effect that might have been experienced as a result of the here-and-now decisions

Stochastic programming with recourse

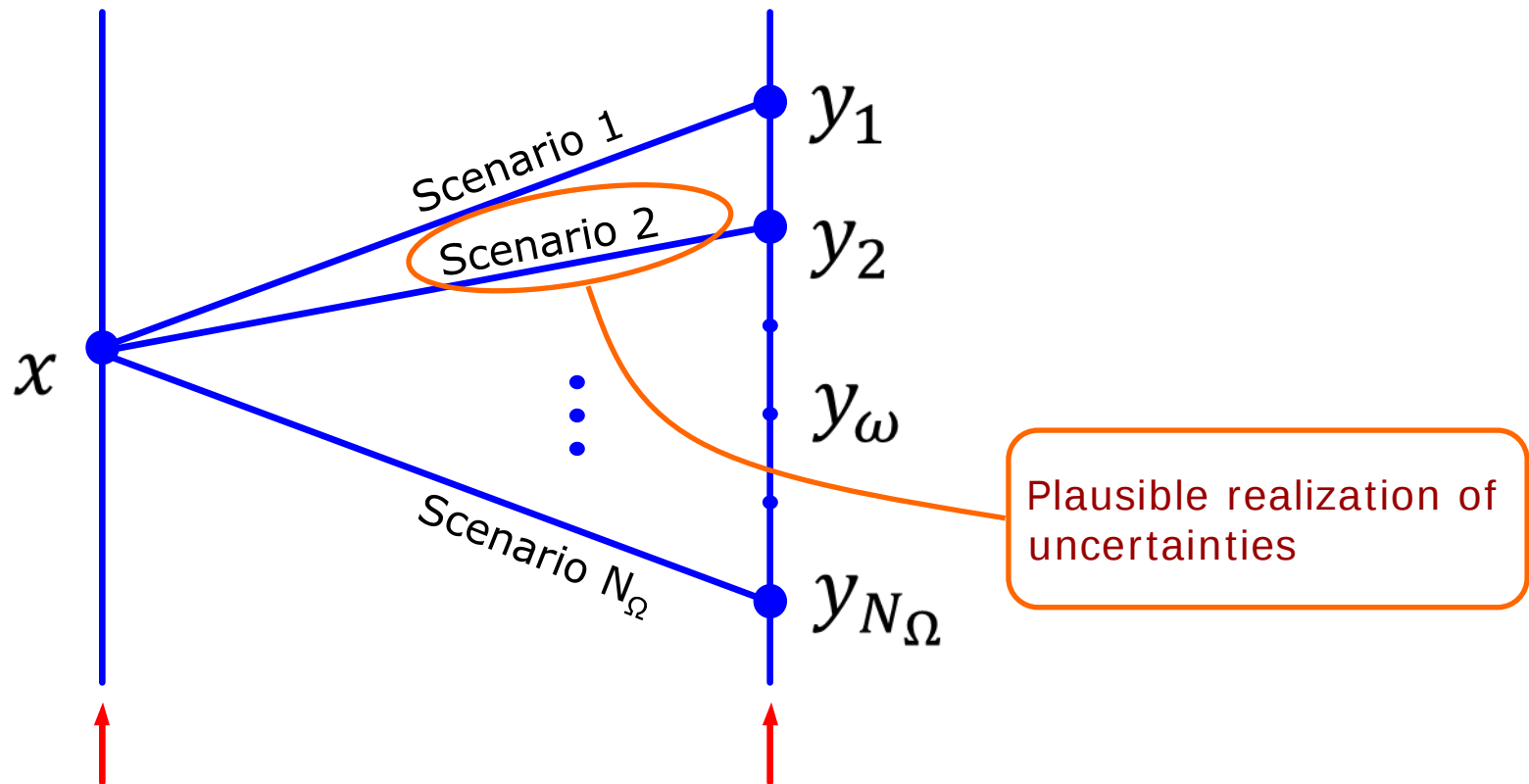


Scenario tree

Here-and-now decisions
First-stage decisions

Wait-and-see decisions
Second-stage decisions
Recourse decisions

Stochastic programming with recourse

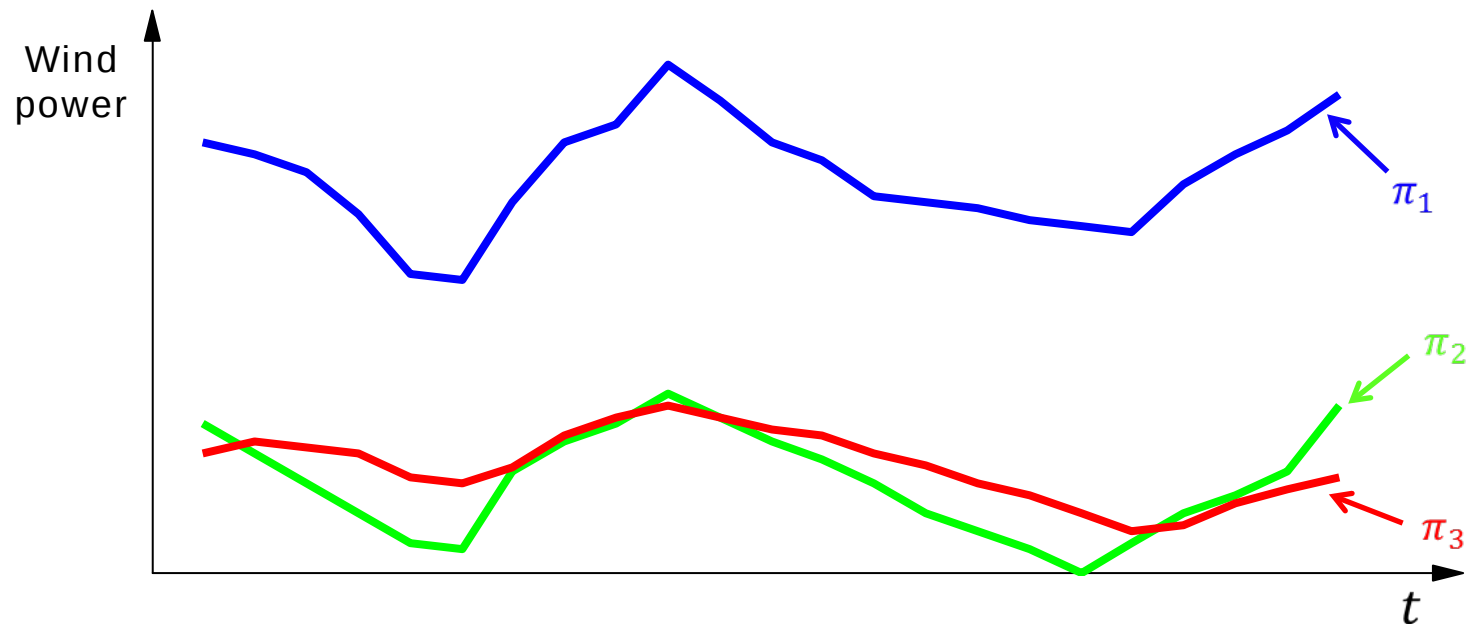


Here-and-now decisions
First-stage decisions

Wait-and-see decisions
Second-stage decisions
Recourse decisions

Stochastic programming with recourse

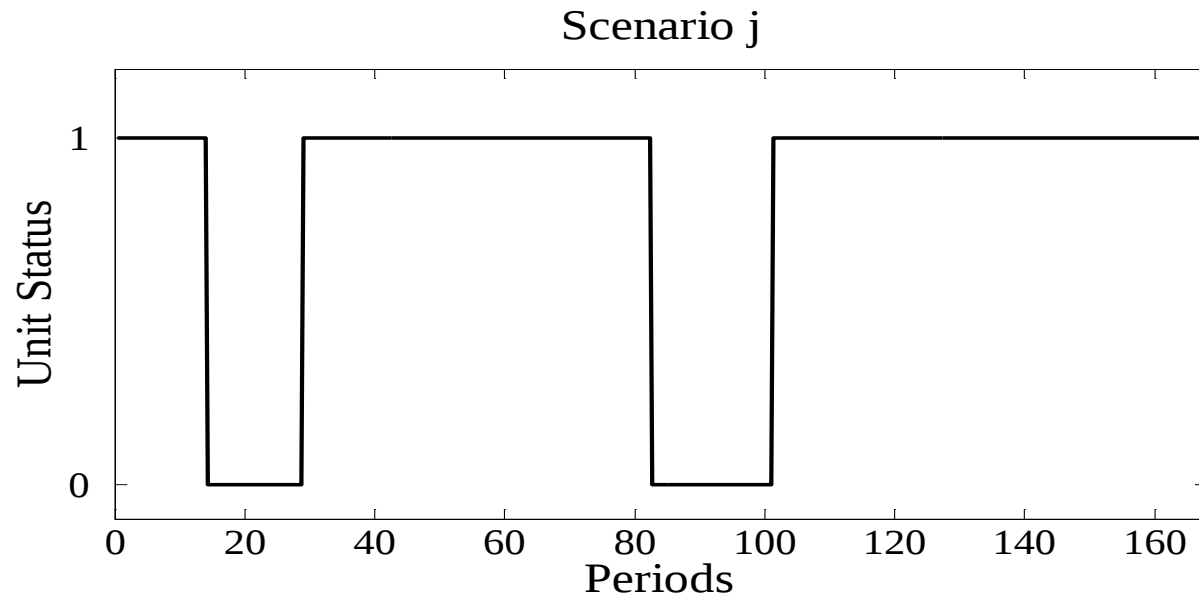
Plausible time trajectories ...



π_ω : Probability of occurrence

Stochastic programming with recourse

The probability distributions governing the data are assumed to be known!



[1 0 0 1 0 1 1 1 ... 0 1 1]

Scenario generation and reduction techniques are imperative!

Stochastic programming with recourse

$$\begin{aligned} & \underset{x}{\text{Maximize}} \quad c^T x + E_{\omega}\{Q(x, \omega)\} \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

being

$$Q(x, \omega) = \left\{ \underset{y_{\omega}}{\text{Maximize}} \quad q(\omega)^T y_{\omega} \right. \\ \left. \begin{aligned} \text{s.t.} \quad & Wy_{\omega} \leq h_{\omega} - T_{\omega}x \\ & y_{\omega} \geq 0 \end{aligned} \right\}, \forall \omega \in \Omega$$

Stochastic programming with recourse

Here-and-now/first-stage decisions

$$\begin{aligned} & \underset{x}{\text{Maximize}} \quad c^T x + \mathbb{E}_\omega \{Q(x, \omega)\} \\ & \text{s.t.} \quad Ax \leq b \\ & \quad \quad x \geq 0 \end{aligned}$$

being

Wait-and-see/recourse/second-stage decisions

$$\begin{aligned} Q(x, \omega) = \left\{ \underset{y_\omega}{\text{Maximize}} \quad q(\omega)^T y_\omega \right. \\ \left. \text{s.t.} \quad Wy_\omega \leq h_\omega - T_\omega x \right. \\ \left. \quad \quad y_\omega \geq 0 \right\}, \forall \omega \in \Omega \end{aligned}$$

Stochastic programming with recourse

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + E_{\omega}\{Q(x, \omega)\} \\ \text{s.t.} &&& Ax \leq b \\ &&& x \geq 0 \end{aligned}$$

being

Recourse problem

$$Q(x, \omega) = \left\{ \underset{y_{\omega}}{\text{Maximize}} \quad q(\omega)^T y_{\omega} \right. \\ \left. \begin{array}{l} \text{s.t.} \\ Wy_{\omega} \leq h_{\omega} - T_{\omega}x \\ y_{\omega} \geq 0 \end{array} \right\}, \forall \omega \in \Omega$$

Stochastic programming with recourse

$$\text{Maximize}_{x} c^T x + E_{\omega}\{Q(x, \omega)\}$$

$$\text{s. t.} \quad \begin{aligned} Ax &\leq b \\ x &\geq 0 \end{aligned}$$

being

Feasibility is guaranteed for every scenario
(every possible state of nature)

$$Q(x, \omega) = \left\{ \begin{aligned} &\text{Maximize}_{y_{\omega}} q(\omega)^T y_{\omega} \\ &\text{s. t.} \quad \boxed{Wy_{\omega} \leq h_{\omega} - T_{\omega}x} \\ &\quad y_{\omega} \geq 0 \end{aligned} \right\}, \forall \omega \in \Omega$$

Stochastic programming with recourse

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + \mathbb{E}_\omega \{Q(x, \omega)\} \\ & \text{s.t.} && Ax \leq b \\ & && x \geq 0 \end{aligned}$$

being

Function that takes a random variable and returns a real value

$$Q(x, \omega) = \left\{ \underset{y_\omega}{\text{Maximize}} \quad q(\omega)^T y_\omega \right. \\ \left. \begin{array}{l} \text{s.t.} \\ Wy_\omega \leq h_\omega - T_\omega x \\ y_\omega \geq 0 \end{array} \right\}, \forall \omega \in \Omega$$

Stochastic programming with recourse

$$\begin{array}{ll}
 \text{Maximize}_{x} & c^T x + E_{\omega}\{Q(x, \omega)\} \\
 \text{s.t.} & Ax \leq b \\
 & x \geq 0
 \end{array}$$

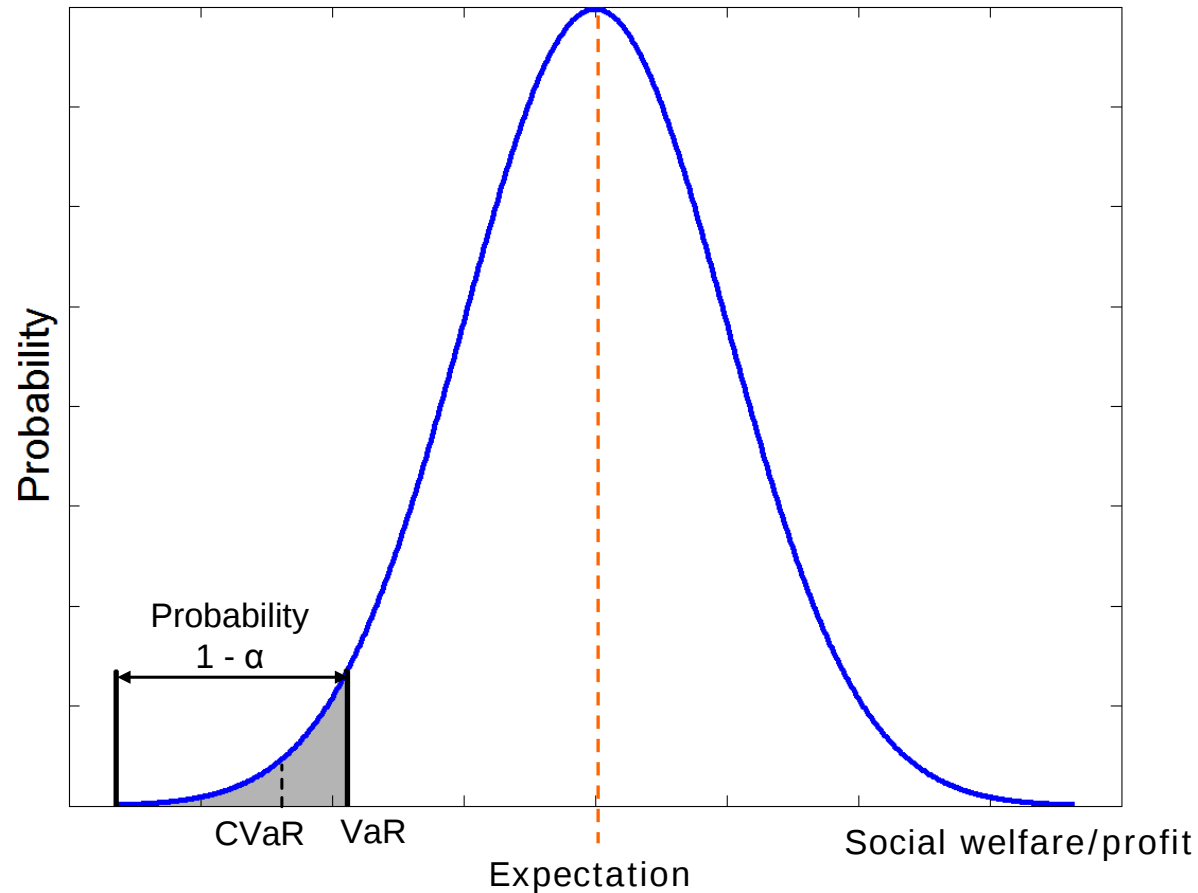
being

Function that takes a random variable and returns a real value

- Expected value
- Any other quantile

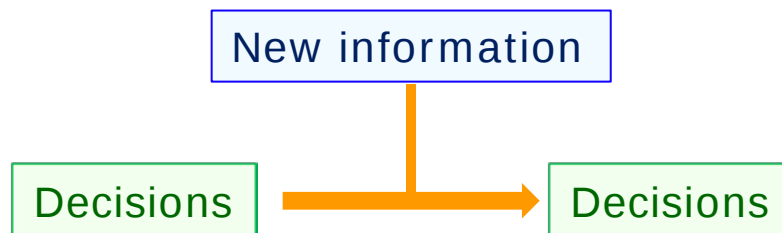
$$Q(x, \omega) = \left\{ \begin{array}{ll} \text{Maximize}_{y_{\omega}} & q(\omega)^T y_{\omega} \\ \text{s.t.} & W y_{\omega} \leq h_{\omega} - T_{\omega} x \\ & y_{\omega} \geq 0 \end{array} \right\}, \forall \omega \in \Omega$$

Stochastic programming with recourse



Stochastic programming with recourse

Two-stage setting:



Stochastic programming with recourse

Multi-stage setting:



Chance-constrained programming approach

$$\begin{array}{ll}\text{Maximize} & c^T x \\ \text{s. t.} & \boxed{P[Ax \leq b] \geq p} \\ & x \geq 0\end{array}$$

Chance constraint

$$P[H(x, \omega) \leq 0] \geq p$$

Robust optimization approach

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + G(x, \omega) \\ \text{s.t.} &&& Ax \leq b \\ &&& x \geq 0 \end{aligned}$$

being

$$G(x, \omega) = \left\{ \begin{array}{ll} \underset{\omega \in \Omega}{\text{Min}} & \underset{y_\omega}{\text{max}} \quad q^T y_\omega \\ \text{s.t.} & Wy_\omega \leq h_\omega - T_\omega x \\ & y_\omega \geq 0 \end{array} \right\}, \forall \omega \in \Omega$$

Robust optimization approach

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + G(x, \omega) \\ \text{s.t.} &&& Ax \leq b \\ &&& x \geq 0 \end{aligned}$$

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$$G(x, \omega) = \left\{ \underset{\omega \in \Omega}{\text{Min}} \max_{y_\omega} q^T y_\omega \right. \\ \left. \text{s.t.} \quad \begin{aligned} & Wy_\omega \leq h_\omega - T_\omega x \\ & y_\omega \geq 0 \end{aligned} \right\}, \forall \omega \in \Omega$$

Worst-case scenario!

The solution remains feasible, thus *robust*, for any realization of the uncertain parameters

Robust optimization approach

$$\begin{aligned} & \underset{x}{\text{Maximize}} && c^T x + G(x, \omega) \\ \text{s.t.} &&& Ax \leq b \\ &&& x \geq 0 \end{aligned}$$

being

Just the support (range) of uncertain parameters is needed

$$G(x, \omega) = \left\{ \underset{\omega \in \Omega}{\text{Min}} \max_{y_\omega} q^T y_\omega \right. \\ \left. \text{s.t.} \quad \begin{aligned} Wy_\omega &\leq h_\omega - T_\omega x \\ y_\omega &\geq 0 \end{aligned} \right\}, \forall \omega \in \Omega$$

A rough comparison ...

	Stochastic programming with recourse	Chance-constrained programming	Robust optimization
Recourse structure?	Yes	No	Not necessarily
Uncertainty model?	Probability distribution	Probability distribution	Deterministic set
Scenarios?	Usually	Not necessarily	No
Feasible?	For every plausible state of nature	"As much as possible"	Immune to uncertainty
Optimal?	Quantile target	Tradeoff: optimality vs. feasibility	Worst-case scenario

To gain insight into ...

Stochastic programming

- J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*. New York: Springer-Verlag, 1997

Robust optimization

- D. Bertsimas and M. Sym, "The price of robustness," *Operations Research*, vol. 52, no. 1, pp. 35–53, 2004

Decision making in electricity markets

- A. J. Conejo, M. Carrión, J. M. Morales, *Decision Making Under Uncertainty in Electricity Markets*, International Series in Operations Research & Management Science, Springer, New York. 2010

Example

Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Demand	150	125	300
Price (\$/unit)	60	40	10

Example

Resource	Desk
Lumber (\$2/item)	8
Finishing (\$4/h)	4
Carpentry (\$5.2/h)	2
Demand	150
Price (\$/unit)	60

$$\text{Cost} = 8 \cdot 2 + 4 \cdot 4 + 2 \cdot 5.2 = \$42.4/\text{unit}$$

$$\text{Profit} = 60 - 42.4 = \$17.6/\text{unit}$$

Example

Resource	Desk
Lumber (\$2/item)	8
Finishing (\$4/h)	4
Carpentry (\$5.2/h)	2
Demand	150
Price (\$/unit)	60
Cost (\$/unit)	42.4
Profit (\$/unit)	17.6

Example

Resource	Desk	Table	Chair
Lumber (\$2/item)	8	6	1
Finishing (\$4/h)	4	2	1.5
Carpentry (\$5.2/h)	2	1.5	0.5
Demand	150	125	300
Price (\$/unit)	60	40	10
Cost (\$/unit)	42.4	27.8	10.6
Profit (\$/unit)	17.6	12.2	-0.6

Example

x_l = number of lumber items

x_f = number of labor hours for finishing

x_c = number of labor hours for carpentry

y_d = number of desks produced

y_t = number of tables produced

y_c = number of chairs produced

$$\text{Max } 60y_d + 40y_t + 10y_c - 2x_l - 4x_f - 5.2x_c$$

s.t.

$$y_d \leq 150$$

$$y_t \leq 125$$

$$y_c \leq 300$$

$$8y_d + 6y_t + y_c \leq x_l$$

$$4y_d + 2y_t + 1.5y_c \leq x_f$$

$$2y_d + 1.5y_t + 0.5y_c \leq x_c$$

Example

Resource	Desk	Table	Chair
Demand	150	125	300
Price (\$/unit)	60	40	10
Cost (\$/unit)	42.4	27.8	10.6
Profit (\$/unit)	17.6	12.2	-0.6
Production	150	125	0

The company needs 1950 lumber items, 850 labor hours for finishing and 487.5 labor hours for carpentry

Example

- Maybe we do not know the exact demand values.

DEMAND	Low value	Most likely value	High value	Expected value
Desk	50	150	250	150
Tables	20	110	250	125
Chairs	200	225	500	300
Probability	0.3	0.4	0.3	-

How to solve the problem?

- Solutions:
 - Use the expected demand and proceed (this is what we just did!)
 - Solve the problem for each demand scenario and somehow combine the solutions

Example

DEMAND		Low value	Most likely value	High value	Expected value
Resource quantities	Lumber	520	1860	3500	1950
	Finishing	240	820	1500	850
	Carpentry	130	465	875	487.5
Production quantities	Desk	50	150	250	150
	Tables	20	110	250	125
	Chairs	0	0	0	0
Profit (\$)		1124	3982	7450	4165

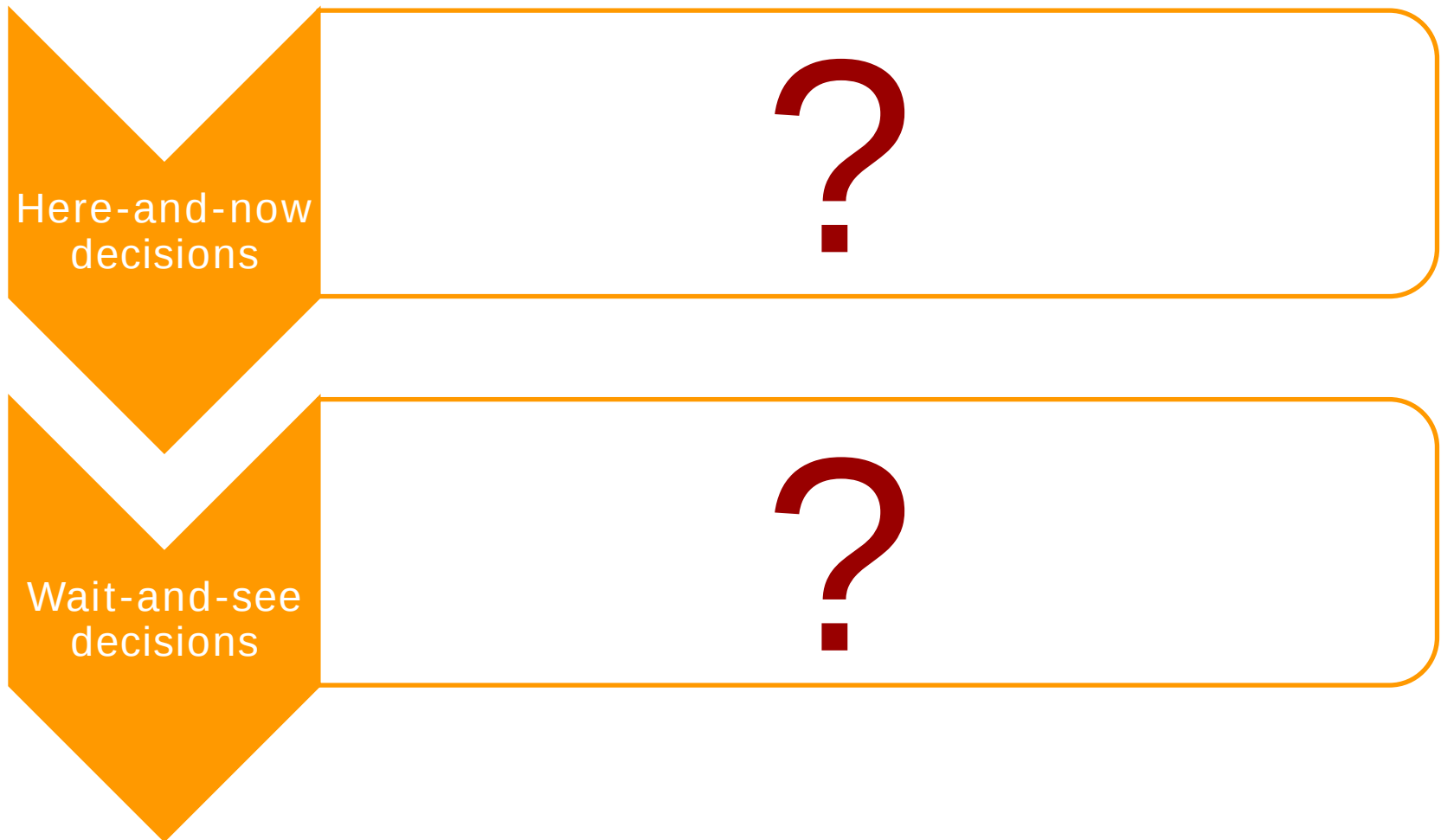
Example

- We need a model that explicitly considers the various scenarios... not individually, but collectively: **stochastic solution**.
- Any suggestion?

Example

- We need a model that explicitly considers the various scenarios... not individually, but collectively: **stochastic solution**.
- **Recourse model**: some decisions must be made before the realization of stochastic parameters, while some of them can be delayed until afterward.

Example



Example

Here-and-now decisions

- Number of lumber items
- Number of labour hours for finishing
- Number of labour hours for carpentry

Wait-and-see decisions

- Number of desks produced
- Number of tables fabricated
- Number of chairs produced

Example

Here-and-now
decisions

- Number of lumber items
 - Number of labour hours for finishing
 - Number of labour hours for carpentry
- X

Wait-and-see
decisions

- Number of desks produced
 - Number of tables fabricated
 - Number of chairs produced
- Y

Example

x_l = number of lumber items

x_f = number of labor hours for finishing

x_c = number of labor hours for carpentry

$y_{d\omega}$ = number of desks produced in scenario ω

$y_{t\omega}$ = number of tables produced in scenario ω

$y_{c\omega}$ = number of chairs produced in scenario ω

Example

$$\underset{x_b, x_f, x_c}{\text{Maximize}} -2x_b - 4x_f - 5.2x_c + E_{\omega}\{Q(x, \omega)\}$$

$$\text{s.t. } x_b, x_f, x_c \geq 0$$

$$Q(x, \omega) = \left\{ \underset{y_{d\omega}, y_{t\omega}, y_{c\omega}}{\text{Maximize}} \quad 60y_{d\omega} + 40y_{t\omega} + 10y_{c\omega} \right.$$

$$8y_{d\omega} + 6y_{t\omega} + y_{c\omega} - x_b \leq 0$$

$$4y_{d\omega} + 2y_{t\omega} + 1.5y_{c\omega} - x_f \leq 0$$

$$2y_{d\omega} + 1.5y_{t\omega} + 0.5y_{c\omega} - x_c \leq 0$$

$$0 \leq y_{d\omega} \leq d_{d\omega}$$

$$0 \leq y_{t\omega} \leq d_{t\omega}$$

$$0 \leq y_{c\omega} \leq d_{c\omega} \}, \forall \omega$$

$$E_{\omega}\{Q(x, \omega)\} = \sum_{\omega} p_{\omega} (60y_{d\omega} + 40y_{t\omega} + 10y_{c\omega})$$

Example

DEMAND		Low value	Most likely value	High value	Expected value	Stochastic solution		
Resource quantities	Lumber	520	1860	3500	1950	1300		
	Finishing	240	820	1500	850	540		
	Carpentry	130	465	875	487.5	325		
						Demand		
						Low	Most likely	High
Production quantities	Desk	50	150	250	150	50	80	80
	Tables	20	110	250	125	20	110	110
	Chairs	0	0	0	0	200	0	0
Profit (\$)		1124	3982	7450	4165	1730		

Example

DEMAND		Low value	Most likely value	High value	Expected value	Stochastic solution
Resource quantities	Lumber	520	1860	3500	1950	1300
	Finishing	240	820	1500	850	540
	Carpentry	130	465	875	487.5	325
Expected Profit (\$)		1124	1702	-2050	1545	1730

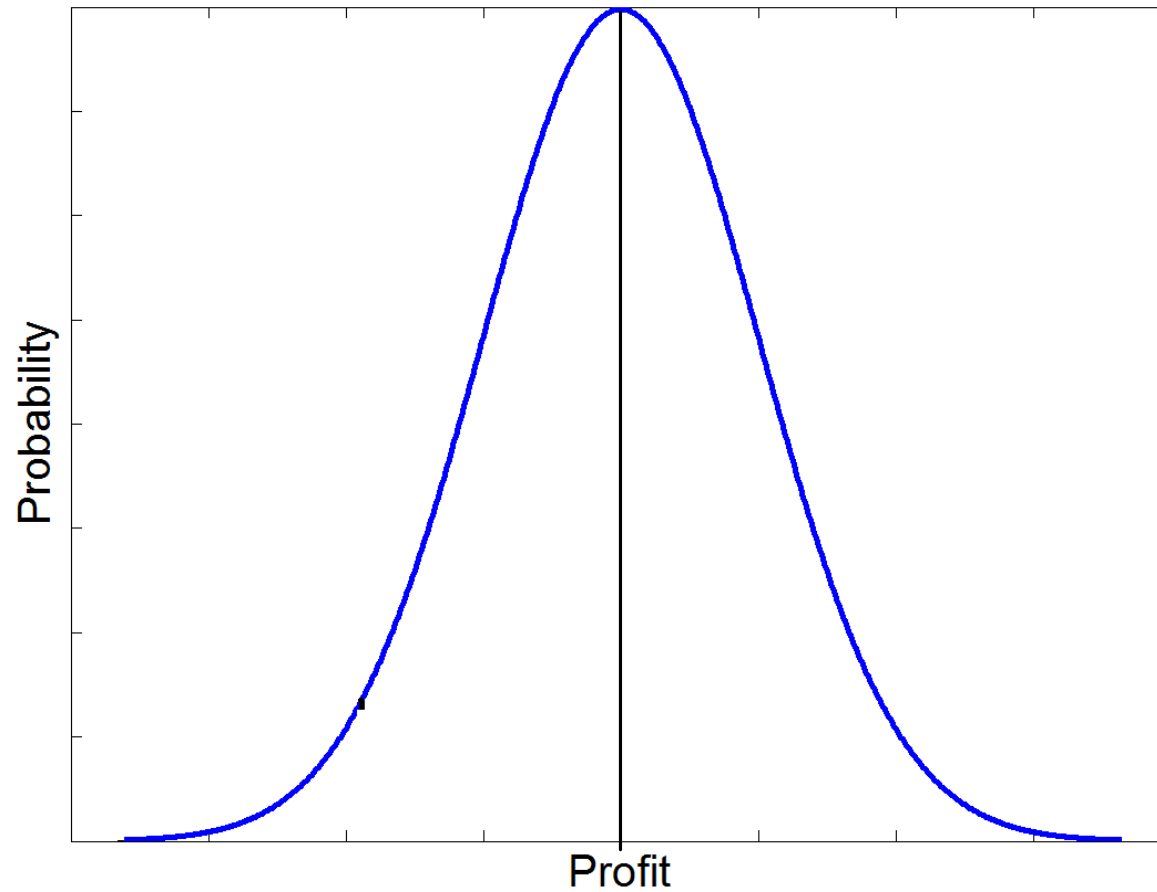
Example

- Advantages:
 - Resource provision considers all possible scenarios to maximize the expected profit.

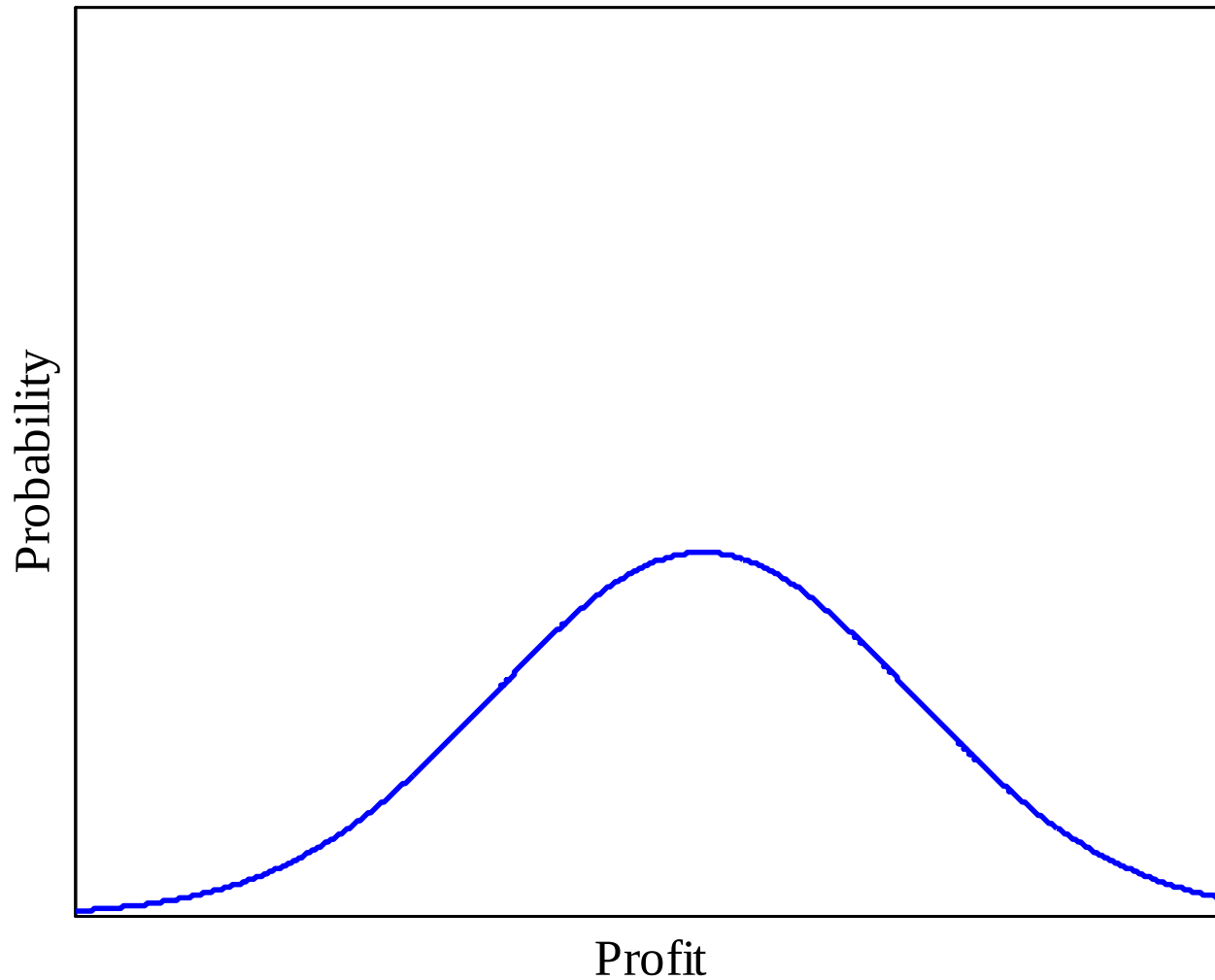
Example

- Advantages:
 - Resource provision considers all possible scenarios to maximize the expected profit.
 - A risk measure could be included easily.

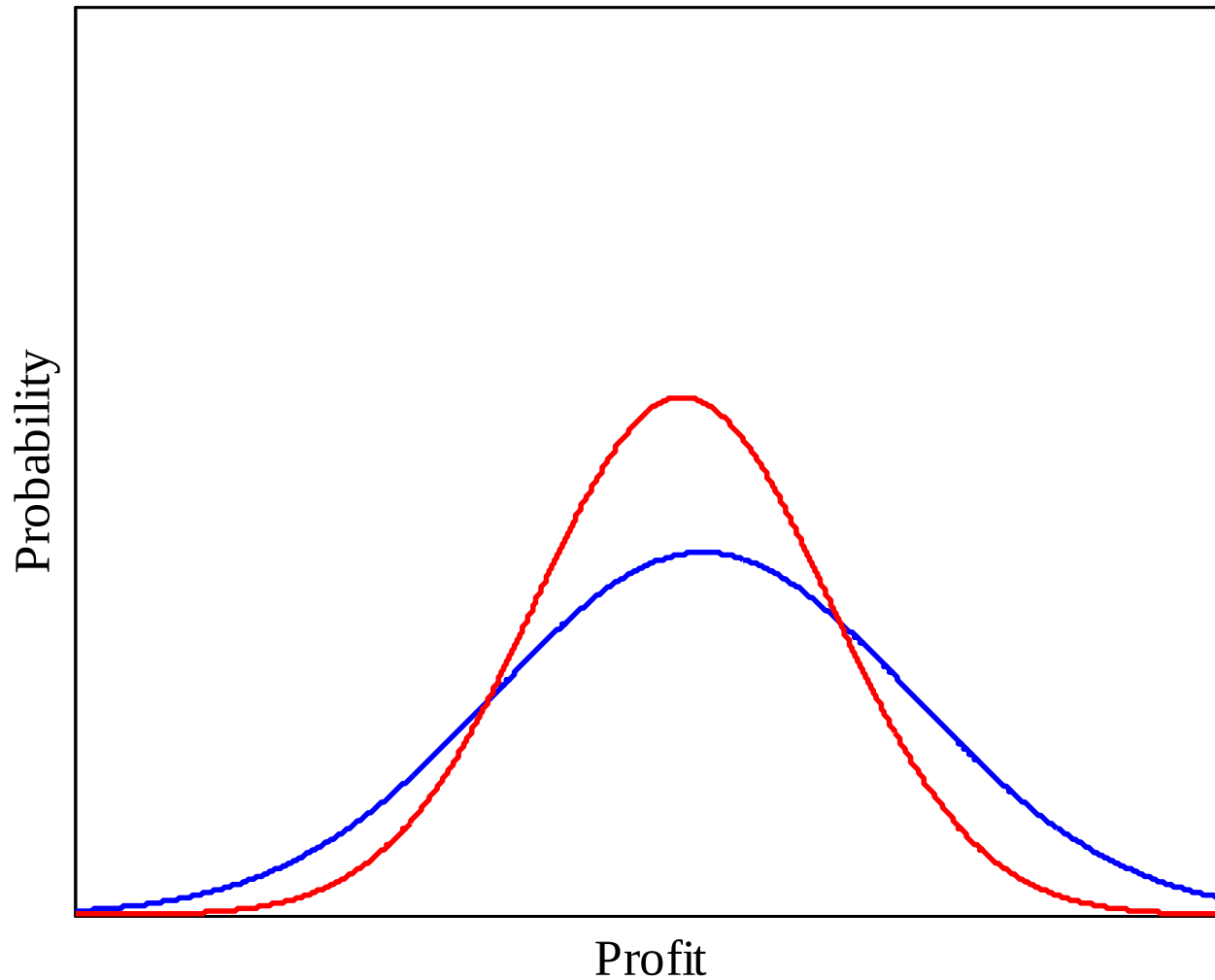
Example



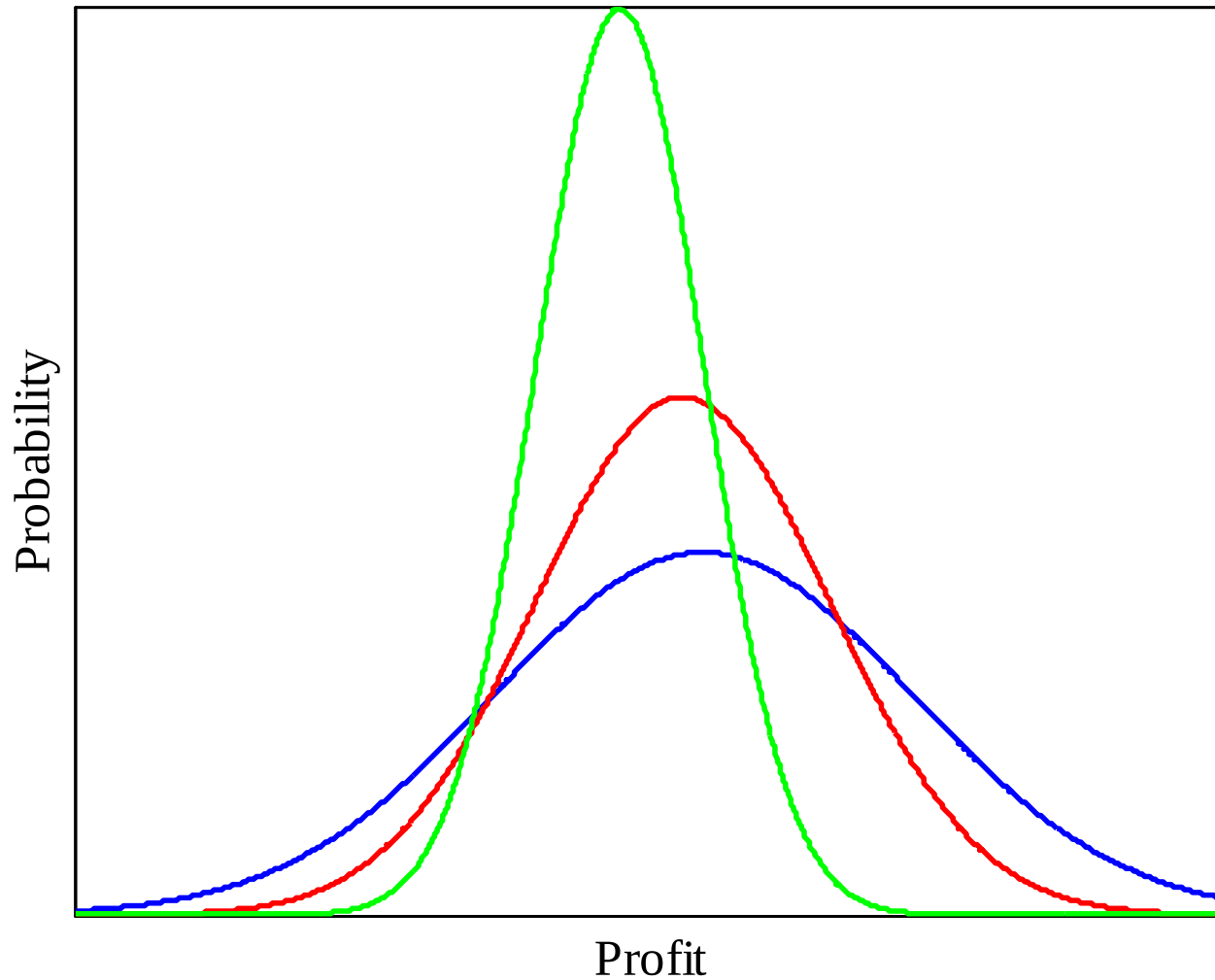
Example



Example



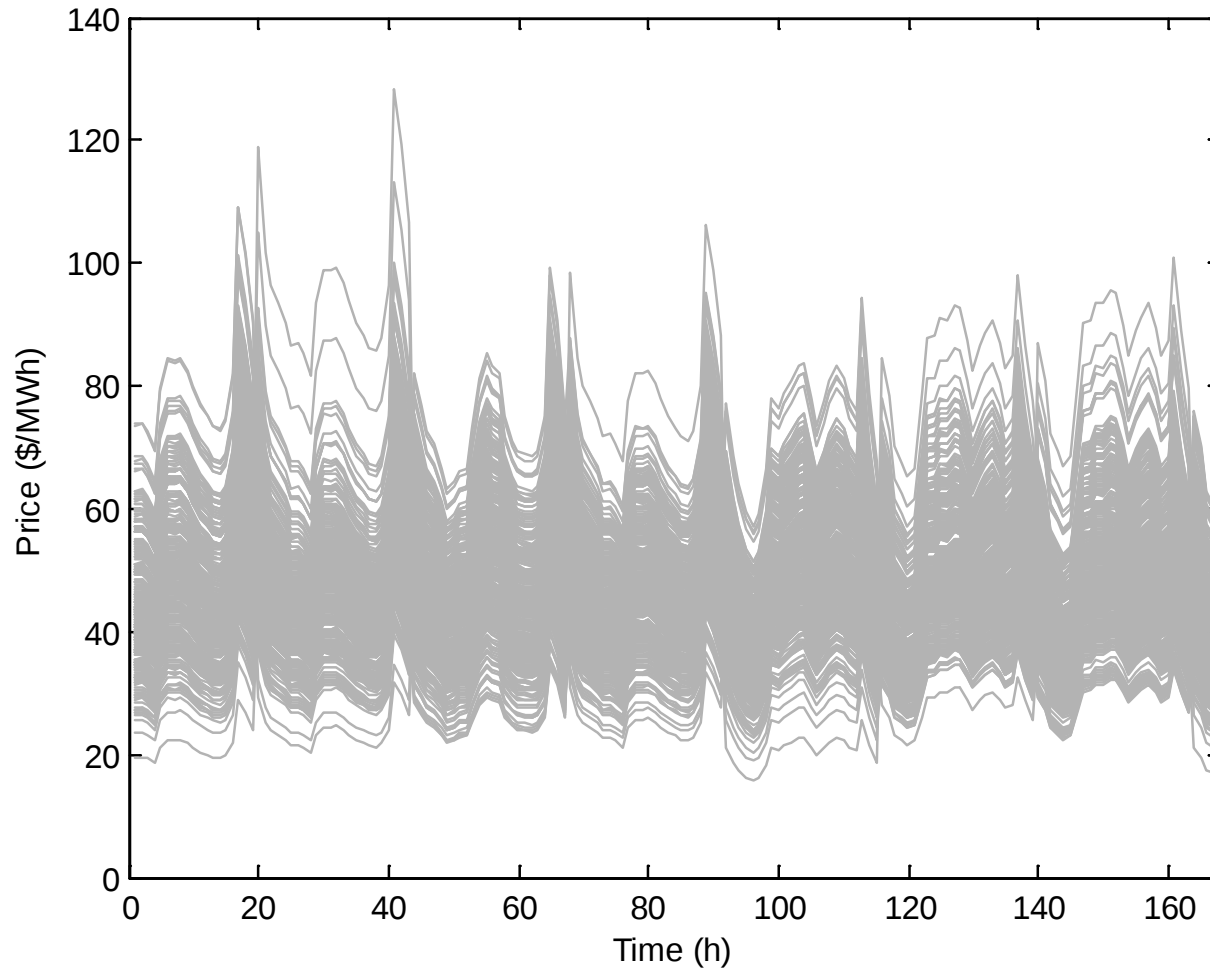
Example



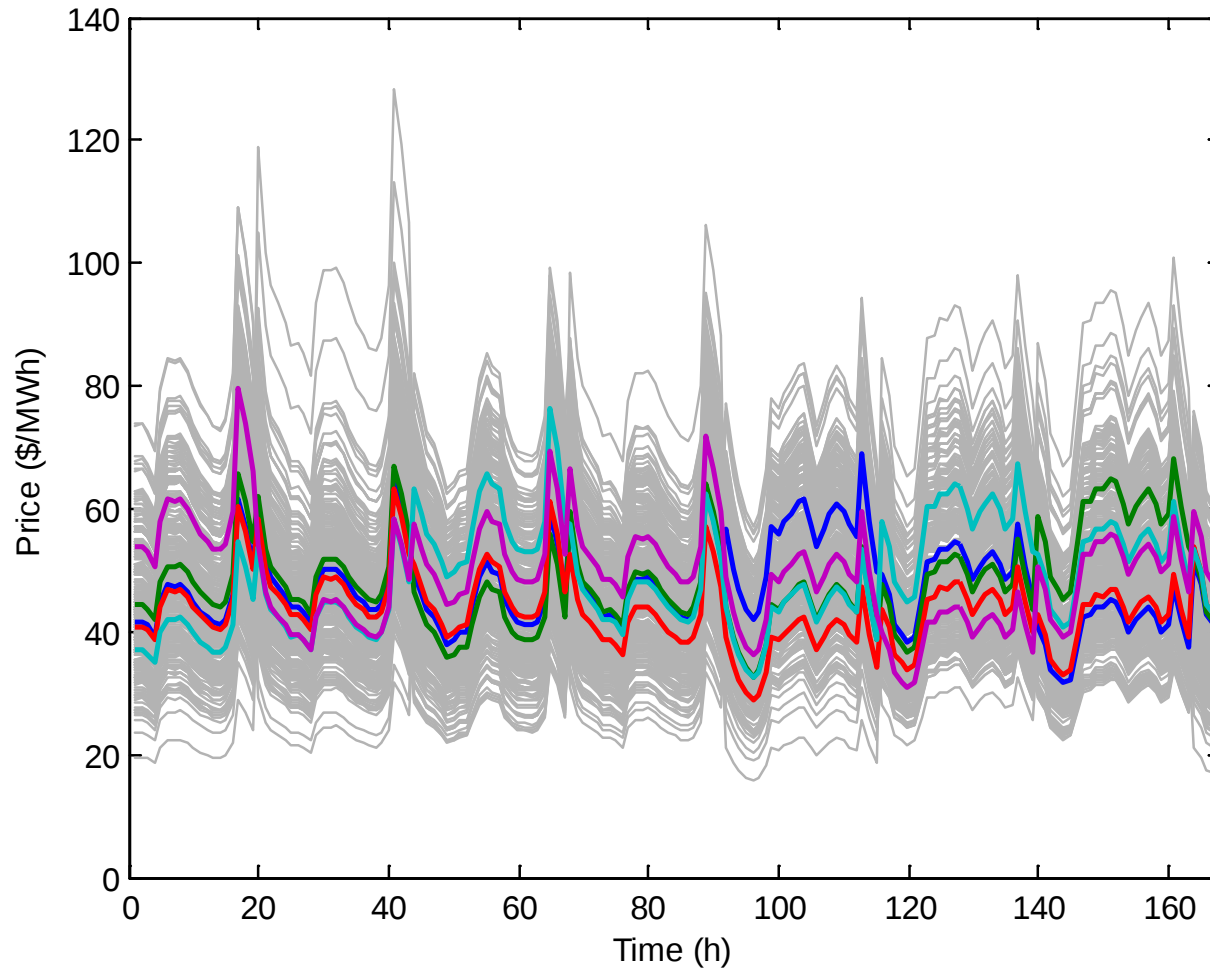
Stochastic programming

- Advantages:
 - Resource provision considers all possible scenarios to maximize the expected profit.
 - A risk measure could be included easily.
- Disadvantages:
 - The size of the model increases with the number of scenarios (scenario reduction).

Scenario reduction



Scenario reduction





Thanks for your attention!